An approach to multi-objective robust optimization allowing for explicit analysis of robustness

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Robust Optimization and Probabilistic Analysis of Robustness (ROPAR)

Consider a model y = M (x, p).
x(t)= uncertain input (time series, t=1,T), p=parameters (decision variables), M=model, y(t)=model output (time series t=1,T), OFk=objective functions indicating some quality of the model to be minimized (k=1,K). If M is a hydrological model, the objective function could the error of the model with respect to measured ym(t) measured by RMSE for t=1,T. If M is flood assessment model (based on running a hydraulic model to calculate overflows), then the objective functions may represent the calculated flood damage and costs.

Deterministic optimization problem DOP:
If K=1: Find vector p* such that OF=min.
If K>1: Find a set of parameters vectors p*s mapped to a Pareto set PS in objective space {OF1…OFK}.

This can be done easily for deterministic x(t). If however we assume that values of x(t) are uncertain, then the found p* for different values of x(t) may not lead to minimum OF.

Here we define *robust optimization* as a problem of finding such p** that OF would not change much for different (samples) realizations of x(t).

Here we define the problem of *analysis of robustness* as a problem of analyzing how much the optimal solution changes in case of using different (samples) realizations of x(t).

An algorithm for *Robust Optimization and Probabilistic Analysis of Robustness (ROPAR)* follows:

1. for i = 1 to IMAX do
begin
   2. Sample time series xi(t), t=1,T
   3. Find p*i by solving the deterministic optimization problem DOP. It results also in the Pareto set PSi.
end.
4. Pick one objective function, say k=k1, to fix. Choose a value OFk1=OFk1fix.
5. Pick another objective function for analysis, say k=k2.
6. For OF2k2 consider all found solutions in object functions space {OF1…OFK} for all i=1,IMAX, and build a frequency diagram for the values of OFk2.
This will be an approximation of probability density function characterising the uncertainty of solutions w.r.t. OFk2. If it is wide (e.g. StDev is high), uncertainty of knowing exact value of OFk2 is high (so robustness of optimal solutions is low). If it is narrow (e.g. StDev is low), uncertainty of knowing exact value of OFk2 is low (so robustness of optimal solutions is high).

Variations:
1) Some parameters of model M can be also uncertain, not only x(t).
2) Instead of probabilistic analysis, it can be based on fuzzy characterisation of uncertainty.